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Bank bailouts: moral hazard vs. value effect

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Abstract

The traditional approach to the central bank's lender of last resort function emphasizes the trade-off between being too 'tough', and thus increasing the likelihood that the failure of a single bank hampers the confidence in the whole banking system, and being too 'soft', thereby creating incentives for banks to take on excessive risk. In contrast with this view, we show that a central bank, by announcing and committing ex-ante to bail out insolvent institutions in times of adverse macroeconomic conditions, can create a risk-reducing 'value effect' that outweighs the moral hazard component of the policy, and thus lowers bank risk.

JEL classification: D82; E44; E58

Keywords: Banks; Risk; Bailout; Lender of last resort; Value effect; Moral hazard

I *paesi e i tempi dei più atroci supplicii furon sempre quelli delle più sanguinose ed inumane azioni*, *Cesare Beccaria*, Dei Delitti e delle Pene (1764)

1. Introduction

Economists (and possibly central bankers) disagree on how liberal the access to last-resort support should be. Those who follow Bagehot's gospel argue that central banks should only assist temporarily illiquid banks.[1] Others, either stressing the negative externalities associated with bank failures (e.g., Solow, 1982), or the difficulties of distinguish-

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[1] For a 'modern' reformulation of the Bagehot's doctrine, see Rochet and Vives (2002).

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ing between illiquidity and insolvency (e.g., Goodhart, 1988, 1995), suggest that the access to the lender of last resort (LOLR) should not be a priori denied to any bank. In general, however, everyone seems to agree that central banks face a trade-off between being too 'tough,' thus increasing the likelihood that the failure of a single bank hampers the confidence in the whole banking system, and being too 'soft,' thereby creating incentives for banks to engage in excessive risk-taking.[2]

To what extent and under what conditions does a soft LOLR approach induce moral hazard and a higher risk appetite? This is the question we address. In this paper, in contrast to the conventional view, we show that the standard argument that LOLR bailouts engender moral hazard may be off the mark. More precisely, we show that a central bank that announces and commits ex-ante to bailing out insolvent institutions in times of adverse macroeconomic conditions beyond the control of bank managers, by increasing the value of the bank charter, creates a risk-reducing 'value effect' that more than offsets the moral hazard component of such a policy. As a result, this contingent bailout policy reduces both the frequency of bankruptcies *and* overall bank risk.

In our framework, where the bank's risk exposure is non-observable to the central bank, bank managers maximize expected profits choosing from a menu of portfolios with different risk-return profiles. The bank's decision entails the following trade-off: While the choice of riskier portfolios (relative to the one that maximizes expected returns in the absence of limited liability) tends to increase one period bank profits, it also increases the probability that the bank becomes insolvent at the end of the period and loses its charter. In this context, the introduction of a bailout scheme has two mutually offsetting effects. On the one hand, it creates moral hazard, as the probability of surviving depends less on the bank's choice of risk and more on the central bank's actions. On the other hand, it increases the bank's probability of survival, thus raising the charter value at stake in case of failure and, in turn, the bank's incentives to protect it by choosing safer investments.

Whereas in general the net result of these two influences can move in either direction, we find that there is always a publicly announced policy that commits to bailing out failed institutions only in the event of an adverse macroeconomic shock that indeed yields a lower equilibrium level of risk. The intuition for the result is easy to grasp if we consider the case of a catastrophic event such as a war or an earthquake. Since in those cases banks are likely to become insolvent irrespective of their portfolio choice, an insurance contract that protects them from catastrophes will elicit little, if any, additional moral hazard-related risk-taking.[3] On the other hand, insurance, by reducing the probability of bankruptcy in those states, increases the banks' value and, through this channel, their incentives to limit their overall risk exposure.

Among the few formal models of bank bailouts in the literature,[4] perhaps the one that more explicitly represents the standard social cost-moral hazard trade-off is Goodhart and Huang (1999). They show that the central bank has incentives to provide LOLR assistance

[2] See Bordo (1989) and Giannini (1998) for more exhaustive discussions of LOLR theory and practice.

[3] More in general, when measured in terms of the probability of success of the bank, the distinction between safe and risky portfolios tends to vanish whenever the economy faces a large systemic shock.

[4] Our review of the literature is very selective. We refer to Battacharya and Thakor (1993) and Freixas and Rochet (1997) for a more detailed discussion of the contributions in the area.

inasmuch as concerns about bank contagion are weighted more strongly than moral hazard considerations. Moreover, the authors argue that, in order to minimize the moral hazard component, it is optimal for the central bank to use discretion ('constructive ambiguity') in the bailout decision. In a related paper, Freixas (1999) finds that depending on the characteristics of the bank's balance sheets, and on the social cost of bank failures, the optimal policy may be either a systematic bailout or a mixed strategy, with the latter providing a theoretical foundation for the constructive ambiguity doctrine.

While, in these two papers, tougher policies have the advantage of limiting moral hazard, some authors have recently argued that a 'soft' policy can be used to induce bank managers to reveal private information about the realization of portfolio returns. Povel (1996), for example, shows that a soft policy can induce an entrepreneur protected by limited liability to reveal at an early stage that his firm is in financial distress, thus allowing for a less expensive rescue. A similar idea, for the case of bank bailouts, is proposed by Aghion et al. (1999). Mailath and Mester (1994) look instead at the incentives that a distressed bank has in investing in excessively risky projects and show that when regulators cannot commit to future actions, then forbearance arises as an equilibrium outcome. This result is in the spirit of Boot and Thakor (1993). They look at the problems induced by regulators' reputational incentives and find that a reputation-seeking regulator will tend to delay bank closure relative to the optimum. On these grounds they favor a rule-based regulation that limits regulatory discretion.

The link between bank value and risk-taking decisions, namely the fact that in a dynamic setting the loss of the charter value may act as a disciplinary device against risk taking, is not new to the banking literature. Herrings and Vankudre (1987) show that the lower is an intermediary's net worth the higher its willingness to take large gambles at the cost of lower total returns. Hellmann et al. (2000) discuss the effect of prudential regulation on banks' risk-taking incentives through their franchise value. Empirical evidence on the disciplinary role of franchise value is provided by Demsetz et al. (1996). Suarez (1994) shows that the threat of being closed can be seen as 'an effective way to induce the bank to be prudent when the present value of its future rents is sufficiently high,' (p. 24) from which he infers that the optimal strategy for the central bank is to credibly commit to withdrawing the bank's charter in case of bankruptcy. Along the same lines, Blum (1999) shows that, in a multiperiod model, tough policies such as strict capital requirements, by decreasing the value of the bank, may end up increasing risk-taking incentives. In a related paper, Acharya (1996) shows that regulatory forbearance may be optimal if dead-weight losses associated with the closure of a bank are important. However, he does not model the bank's optimization problem, the effect of forbearance on bank value, and the optimal choice of risk.[5]

Freixas et al. (1998) develop a model of interbank lending in which banks face liquidity shocks *à la* Diamond and Dybvig (1983) and solvency shocks *à la* Holmström and Tirole (1998). They show that, when entrepreneurs have interest in investing in an inferior technology (because of moral hazard) and it is not possible to distinguish between liquidity and solvency shocks (because of private information), there is excessive liquidation of banks

[5] Empirical evidence supporting the link between value and risk is reported in Keeley (1990).

in the absence of central bank intervention. In their model, the role of LOLR assistance is that of mutualizing the solvency shocks by taxing the 'lucky' banks, and subsidizing the 'unlucky' ones.

The idea of subsidizing 'unlucky' banks is also present in our paper, since bank managers are 'pardoned' if the bank fails in bad states of nature when the bank's fate is relatively less dependent on the manager's decisions. This idea is somewhat similar to Holmström's (1979) optimal contract in which a "repairman receives higher pay if it is found that the failure was outside his control than if it is found that a component that he controls failed" (p. 83). While in his paper the first best contract cannot be written because agents are not risk neutral,[6] in ours moral hazard cannot be avoided, despite the fact that all agents are risk neutral, because of the bankers' limited liability.

The problems of the limited liability constraint in the optimal contract literature was first analyzed by Sappington (1983). Closer to our framework is Innes (1990), in which agents choose their actions before observing the state of nature. He shows that the optimal contract is one in which the agent receives maximal reward when the result is good and maximal penalty when the result is bad. Financial contracts of this type are not frequently seen in practice, and the idea of subsidizing banks when profits are high would take us far away from the bailout issue addressed in this paper.[7]

Our results are consistent with historical LOLR practice. In particular, they appear to motivate, in a simple way, the tendency to provide subsidized funds to problem institutions during crisis periods documented in the empirical evidence.[8] However, there are important differences between our conclusions and what seems to be conventional wisdom concerning bailout policies. First, in contrast with the traditional view, in our framework an explicit commitment to rescue insolvent banks alleviates the associated moral hazard problem, rather than worsen it. Thus, we do not need to introduce a social cost of bank failures to motivate the desirability of a bailout. This, of course, does not imply that we believe these costs to be negligible: If anything, taking them into consideration will reinforce the conclusions of the paper. Second, we show that the 'constructive ambiguity' approach often recommended to attenuate moral hazard, in which the terms of the LOLR arrangement are left to the discretion of the central bank, is always dominated by a policy that commits to rescuing failed banks *with certainty*, conditional upon the realization of an adverse aggregate shock.

In the next section, we present the model. In Section 3 we characterize the optimal bailout policy. Section 4 discusses the main results and their robustness. Finally, Section 5 concludes. All proofs are in Appendix A.

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[6] Harris and Raviv (1979) prove that the problem of moral hazard can be avoided when agents are risk neutral.

[7] Also, as Innes recognizes, if entrepreneurs (banks in our framework) have access to short-term borrowing, they would have incentives to inflate their profits to increase the reward.

[8] The empirical evidence confirms that bailouts of ultimately insolvent banks are the rule rather than the exception. For example, in a cross-country survey of bank failures, Goodhart and Schoenmaker (1995) show that 73 out of their sample of 104 distressed banks ended up being bailed out, while in 21 of the remaining banks depositors' losses were covered virtually in full. Similarly, Santomero and Hoffman (1998) present evidence of the tendency of regulators to delay bank liquidation by granting subsidized funds.

2. The model

Bank managers, acting in the interest of risk-neutral shareholders, collect funds from depositors and invest them in a portfolio with stochastic returns. Depositors have the choice of either depositing in the bank or investing their funds in a risk-free asset offering a gross rate of return $r_f \geqslant 1$. Bank deposits are fully protected by a comprehensive deposit insurance,[9] so that depositors invest in the bank at any posted deposit rate $r \geqslant r_f$. For simplicity, we assume that the aggregate supply of funds in the economy is fixed, and we normalize it to unity.

Bank managers can affect the risk-return profile of their portfolios. More precisely, we assume that a portfolio X offers a gross rate of return $\widetilde{R}$ distributed according to the following two-point distribution:[10]

$$\widetilde{R} = \begin{cases} X & \text{with probability } p(X,\gamma), \\ 0 & \text{with probability } 1 - p(X,\gamma), \end{cases}$$

where $\gamma \in [0,1]$ is a stochastic i.i.d. variable representing a random state of nature, distributed over the interval $[0,1]$ with a continuous density function $f(\gamma)$, and a cumulative distribution function, $F(\gamma)$, $F(1) = 1$. In order to keep our analysis as simple as possible, we impose the following functional form for the probability of success:

$$p(X,\gamma) = \gamma p(X),$$

so that expected returns can be written as

$$E[\widetilde{R}] = \mu p(X)X, \tag{1}$$

with $p(X) \in C^2$ and $\mu \equiv \int_0^1 \gamma f(\gamma)\,\mathrm{d}\gamma < 1$. Accordingly, the higher the realization of γ, the higher the probability of success for any given portfolio choice X. Informally, we will refer to high and low values of γ as 'good' and 'bad' states of nature, respectively.

We assume that a higher X is associated with a lower probability of success p. That is, $p'(X) < 0$. In addition, to avoid corner solutions with infinite risk, we assume that $p''(X) \leqslant 0$, so that (1) is strictly concave in the control variable X, and that there exists a $\overline{X} < \infty$, such that $p(\overline{X}) = 0$. Furthermore, we also assume that $X \geqslant \underline{X} = r_f/\mu$, with $p(\underline{X}) = 1$ and $p'(\underline{X}) > -1/\underline{X}$, so that the risk-free asset is strictly dominated in expected returns by (at least) some risky portfolio.[11] Finally, we assume that the bank's choice of X is not observable so that the central bank cannot implement bailout policies that are contingent on the bank's choice of risk.

The time structure of the model is the following. At time $t = 0$, the central bank announces, and can credibly commit to, a bailout policy that is contingent on the bank's failure (and refuse to recapitalize) in certain states of the world (see below). Given such policy, the bank chooses X_t, at the beginning of each period t, $t \in [0,\infty)$. If at the end of

[9] The relaxation of this assumption strengthens our results, as discussed in Section 4.

[10] The stochastic structure of the model is similar to Blum (1999), to which we add a random state of nature limiting the bank's control over risk.

[11] This assumption ensures that some investment portfolio is socially optimal.

the period the portfolio's realized return $\widetilde{R}_t$ is positive, profits are consumed by the bank's shareholders. If $\widetilde{R}_t$ is equal to zero, shareholders decide whether or not to recapitalize the bank. If they decide not to, then all deposit liabilities are transferred to the deposit insurance fund which pays depositors in full. At that point, the central bank will enforce the bailout policy announced in period 0, and decide whether or not to withdraw the bank license accordingly. The structure of the model is recursive: In case the charter is not withdrawn, the bank faces the same problem at the beginning of period $t+1$. Finally, we assume that there are no bankruptcy costs.[12]

2.1. *Optimal risk*

Before turning to the bank's problem and the effects of alternative bailout policies on the manager's choice of risk, we derive the (optimal) solution from the point of view of a central planner. The problem that the central planner faces in each time period can be written as

$$V_c \equiv \max_x \mu p(X)X. \tag{2}$$

From the necessary and sufficient first-order condition, we have that the optimal choice of X, henceforth denoted by X^*, is the unique solution to

$$p'(X)X + p(X) = 0,$$

an expression that can be rewritten as

$$\eta(X) \equiv \frac{p'(X)X}{p(X)} = -1, \tag{3}$$

which tells us that, at the first best solution, the elasticity of the probability of success with respect to the portfolio return in case of success equals minus one. It also tells us that, since

$$\frac{\partial \eta(X)}{\partial X} = \frac{[p''(X)X + p'(X)]p(X) - [p'(X)]^2 X}{[p(X)]^2} < 0, \tag{4}$$

excessive risk (that is, a risk level above the optimum) is associated with $\eta(X) < -1$. Furthermore,

$$\eta(\underline{X}) = \frac{p'(\underline{X})\underline{X}}{p(\underline{X})} > -1$$

implies that the optimal level of risk is strictly positive, that is, $X^* > \underline{X}$.

2.2. *The bank's problem*

We assume that the bank is competitive on the liability side while it has some market power on the asset side. The bank's problem, which consists in maximizing the discounted

[12] Naturally, the presence of bankruptcy costs would strengthen the case for a bailout policy.

flow of profits V_t, valued at period t, is thus given by

$$V_t = \Pi_t + \delta s_t(\cdot)\Pi_{t+1} + \delta^2 s_t(\cdot)s_{t+1}(\cdot)\Pi_{t+2} + \cdots,$$

where $\delta < 1$ is a discount factor, $s_t(X_t, \gamma_t, \cdot)$ is the bank's probability of survival from period t to period $t+1$, and Π_t denotes current expected profits, that is

$$\Pi_t = \mu p(X_t)(X_t - r), \tag{5}$$

with $r = r_f$. We generically denote the probability of survival by $s(\cdot)$, in order to introduce different bailout policies. Furthermore, to rule out supergames, we restrict our attention to Markov strategies. This in turn implies that the problem that the bank faces is a recursive one. Then, dropping the time subscripts, the bank's value can be rewritten as

$$V = \max_X \frac{\Pi(X)}{1 - \delta s(\cdot)}. \tag{6}$$

In the absence of a bailout policy, the central bank withdraws the bank's charter whenever the bank does not meet its obligations. If, in the event of zero returns, the bank's shareholders choose not to recapitalize the bank (see below), the bank's survival probability is given by

$$s(X, \gamma) = \mu p(X),$$

and (6) can be rewritten as:

$$V_a = \max_X \frac{\mu p(X)(X - r)}{1 - \delta\mu p(X)}, \tag{7}$$

where the subscript a stands for absence of a bailout policy. In this case, the necessary and sufficient first-order condition[13] for the maximization in (6) can be expressed as

$$\eta(X) = -\left[1 - \frac{p(X)}{\theta_a}\right]\frac{X}{X - r}, \tag{8}$$

with $\theta_a \equiv 1/\delta\mu$. Using (3), (4), and (8), and denoting by X_a the (unique) solution to Eq. (8), one can immediately verify that

$$\frac{\mu p(X_a)X_a}{r} \leqslant \frac{1}{\delta} \quad \Rightarrow \quad X_a \geqslant X^*. \tag{9}$$

Accordingly, for a given discount rate, the bank is more prone to engage in excessive risk the lower are the returns of the profit maximizing portfolio relative to those of a risk-free portfolio. Alternatively, for given rates of return, a bank with a stronger preference for the present (a higher discount rate $1/\delta$), tends to invest in riskier portfolios.

Note that (9) apparently suggests that banks may in some cases choose too little risk ($X_a < X^*$). However, it can be shown that in those cases, it is in the interest of bank shareholders to recapitalize the bank and avoid default. More precisely, if at the end of the period, the realized portfolio returns are zero, shareholders still have the option to raise the

[13] See Appendix A.

necessary funds in the capital market, thereby avoiding default.[14] This 'non-default' option will be exercised whenever the value of the charter continues to be positive, that is, when the discounted value of future expected profits under the assumption of recapitalization,

$$V_a = \max_X \frac{\mu p(X)X - r}{1-\delta}, \tag{10}$$

exceeds the stock of outstanding deposit liabilities, so that, in equilibrium:

$$r \leqslant \delta V_a = \delta \frac{\mu p(X_a)X_a - r}{1-\delta}.$$

This condition can be rewritten as

$$\frac{\mu p(X_a)X_a}{r} \geqslant \frac{1}{\delta}. \tag{11}$$

But, if condition (11) holds and shareholders anticipate that the bank never defaults, the optimal strategy is then necessarily that of maximizing expected returns in each time period, in which case the bank's problem coincides with the central planner's. Summarizing:

Lemma 1. *Equilibrium risk is never below the optimum*:

(i) *If $\mu p(X_a)X_a/r < 1/\delta$, the bank selects a portfolio X_a strictly riskier than X^*, and defaults whenever portfolio returns are below deposit liabilities*;
(ii) *If $\mu p(X_a)X_a/r \geqslant 1/\delta$, the bank chooses the optimal portfolio X^*, and never defaults.*

Since we are interested in the case in which, in the absence of a bailout policy, bank failures are possible in equilibrium, we henceforth assume that condition (9) is satisfied.

3. Bailout policies

Assume that the monetary authorities announce (and can credibly commit to) the following policy: In the event that a bank's gross investment returns do not cover outstanding deposit liabilities, with a certain probability β the central bank provides the troubled bank the necessary funds to repay depositors while allowing it to continue its operations.[15] Assume further that this probability is known to be contingent on the realization of the stochastic variable γ, so that the bailout policy can be expressed as a function $\beta(\gamma): \gamma \to [0, 1]$. Note that this formulation is quite general and, in particular, encompasses the 'constructive ambiguity' approach, according to which the central bank should

[14] We implicitly assume that existing or prospective shareholders can indeed provide the needed funds, thus abstracting from cases in which liquidity constrained capital markets fail to rescue illiquid but solvent institutions.

[15] Note that, under the assumption of full deposit insurance, a 'bailout' simply implies that the central bank authorizes the insolvent bank to keep its charter. As discussed in Section 4, the results of the paper can be readily extended to partial or no insurance, in which case the policy is comparable to a standard bank bailout by which the central bank provides the funds to cover the bank's obligations.

retain some discretion as to when, and under what circumstances, a distressed bank should be rescued.

We want to study the effects of the implementation of this policy on the individual banks' risk-taking decision, and the characteristics that the policy should present in order to minimize the impact of its moral hazard component. To simplify the analysis we assume that the bailout policy takes the form of a pure transfer through which the failed bank's (net) liabilities are assumed by the central bank without any requirement to repay the loan, or to contribute additional equity.

As before, for any given bailout policy $\beta(\gamma)$, if in the event of zero portfolio returns no bailout funds are forthcoming, the bank's shareholders will choose to avoid default by raising capital in the capital markets whenever deposit liabilities do not exceed expected future profits. Note that the introduction of a bailout policy automatically raises the value of the bank. Thus, even when condition (9) holds, shareholders may be willing to recapitalize the bank in those cases in which the central bank decides not to bail it out. Therefore, we can now distinguish between two possible scenarios, default and non-default, according to whether or not shareholders let the bank default if the central bank decides not to give the bank a second chance. We proceed to characterize each of these scenarios in turn, determining in each of them the bailout policy that minimizes risk. Then, building on these results, we endogenize the shareholders' decision of whether to recapitalize the bank or not (as a function of the chosen bailout policy) and characterize the risk-minimizing bailout policy. Finally, we explore how this policy compares with the one that maximizes a central bank's objective function that weighs in efficiency and quasi-fiscal outlays.

3.1. Default scenario

The bank's probability of survival s is now given by the sum of the probability of positive portfolio returns plus the probability that, if returns are zero, the central bank comes to the rescue:

$$s = \int_0^1 \left[p(X)\gamma + (1 - p(X)\gamma)\beta(\gamma)\right] f(\gamma)\,\mathrm{d}\gamma,$$

or, rearranging,

$$s = p(X)(\mu - C) + \overline{\beta}, \tag{12}$$

with

$$C \equiv \int_0^1 \gamma\beta(\gamma) f(\gamma)\,\mathrm{d}\gamma, \qquad \overline{\beta} \equiv \int_0^1 \beta(\gamma) f(\gamma)\,\mathrm{d}\gamma,$$

and

$$C < \min\{\mu, \overline{\beta}\}. \tag{13}$$

Inserting (12) into (6), we obtain the following expression for the bank's value:

$$V_d = \max_X \frac{p(X)(X-r)\mu}{1-\delta(p(X)(\mu-C)+\overline{\beta})}, \tag{14}$$

where the subscript d denotes the 'default' scenario in the presence of a bailout policy. Note that on the one hand C, which reduces the effect of the bank's portfolio choice on its probability of survival, detracts from the marginal cost of risk in terms of lower expected future rents, generating moral hazard. On the other hand $\overline{\beta}$, a pure subsidy component independent of the realization of γ, increases expected future rents and, in turn, the bank's incentives to behave safely.

The necessary and sufficient first-order condition[16] for the maximization in (14) can be expressed as

$$\eta(X) = -\left[1-\frac{p(X)}{\theta_d}\right]\frac{X}{X-r}, \quad \text{with } \theta_d\big(\beta(\gamma)\big) \equiv \frac{1-\delta\overline{\beta}}{\delta(\mu-C)} > 1. \tag{15}$$

We denote by X_d the (unique) solution to (15). It follows directly from (4), (8), and (15) that a bailout policy reduces risk if, and only if, $\theta_d(\beta(\gamma)) < \theta_a$, a condition that is satisfied whenever

$$\mu\delta\overline{\beta} > C. \tag{16}$$

This leads us to the following lemma:[17]

Lemma 2. *A necessary condition for a bailout policy to reduce bank risk is that the covariance between the bailout probability β and the realized state γ be negative* ($\mathrm{cov}(\beta(\gamma),\gamma) < 0$).

The Lemma states that in order to reduce the moral hazard component, the bailout rule has to penalize insolvency to a greater degree under good realizations of the shock. This is not surprising since the probability that insolvency is caused by the bank's imprudent risk behavior is proportional to the value of the shock γ.[18] By withdrawing funds in good states of nature, the central bank indeed indirectly punishes the bank's imprudent practices. Conversely, a bailout policy that does not discriminate between insolvency due to negative exogenous shocks, and to the banker's own decision ($\mathrm{cov}(\beta(\gamma),\gamma) = 0$), worsens the moral hazard problem and induces more risk taking.[19] We can thus state that:

Proposition 1. *A state-independent bailout policy, $\beta(\gamma) = \beta$ for all γ, increases the equilibrium level of risk.*

[16] See Appendix A.

[17] We thank Giovanni Dell'Ariccia for suggesting this formulation of the necessary condition.

[18] We come back to this point in Section 4.

[19] This applies, of course, to the particular case of a blanket guarantee such that $\beta(\gamma) = 1$, for all γ. Indeed, the presence of aggregate shocks on which to condition the bailout policy is crucial to the effectiveness of the scheme. It can be readily shown that if the distribution of γ is degenerate, so that the support of $f(\gamma)$ collapses to a single point, a bailout policy can only stimulate risk-taking behavior.

Let us assume, for the moment, that under all feasible bailout policies the charter value of a bank with realized returns equal to zero does not exceed its stock of deposit liabilities (that is, the default condition $r > \delta V_d$ is satisfied), so that in the absence of central bank assistance, shareholders choose to default. In this case, the risk-minimizing bailout policy is characterized by the following lemma.

Lemma 3. *If, under all feasible bailout policies $\beta(\gamma)$, the default condition $r > \delta V_d$ is satisfied at the equilibrium, bank risk is minimized by a policy*

$$\beta_d(\gamma) \equiv \begin{cases} 1 & \text{if } \gamma \leqslant \gamma_d, \\ 0 & \text{if } \gamma > \gamma_d, \end{cases} \quad \text{with } \gamma_d \equiv \left\{\gamma\colon\ \gamma - \frac{1}{\theta_d(\beta_d(\gamma))} = 0\right\}. \tag{17}$$

Furthermore, since $\gamma_d \in (0, 1)$, there always exists a bailout policy $\beta_d(\gamma)$ that reduces bank risk (i.e., $X_d < X_a$).

3.2. Non-default scenario

In the non-default scenario, bank shareholders cover deposit liabilities whenever portfolio returns are zero and bailout funds are not forthcoming, an event that occurs with probability $1 - s$, where s is, as before, given by (12). Accordingly, the bank's value can be expressed as

$$V_n = \max_X \frac{p(X)\mu(X - r) - r[1 - p(X)(\mu - C) - \overline{\beta}]}{1 - \delta}, \tag{18}$$

where the subscript n denotes the 'non-default' scenario in the presence of a bailout policy. In turn, the non-default condition requires that, at the equilibrium, the value of the bank exceeds the stock of deposit liabilities, so that

$$r \leqslant \delta V_n, \tag{19}$$

or

$$r \leqslant \frac{p(X_n)X_n}{\psi}, \quad \text{where } \psi \equiv \frac{1 + \delta p(X_n)C - \delta\overline{\beta}}{\delta\mu}, \tag{20}$$

and X_n denotes the bank's optimal choice of risk under this scenario.

Finally, for any given bailout policy such that the non-default condition (20) is satisfied at equilibrium, the necessary and sufficient first-order condition[20] for the maximization in (18) can be written as

$$\eta(X) = -1 + \frac{p'(X)Cr}{\mu p(X)}. \tag{21}$$

We are now in a position to characterize the risk-minimizing bailout policy in the non-default scenario.

[20] See Appendix A.

Lemma 4. *If we restrict our attention to the set of bailout policies such that the non-default condition* (19) *is satisfied, risk is minimized by the policy*:

$$\beta_n(\gamma) \equiv \begin{cases} 1 & \text{if } \gamma \leqslant \gamma_n, \\ 0 & \text{if } \gamma > \gamma_n, \end{cases} \quad \text{with } \gamma_n \equiv \{\gamma\colon\ r = \delta V_n(\beta_n(\gamma))\}. \tag{22}$$

Comparing Lemmas 3 and 4, we can see that the optimal policy shares the same all-or-nothing nature under both scenarios. Thus, in both cases, the best that the central bank can do to minimize risk is, contingent on the realization of a particular value of γ, either to bail out the bank with probability one or to never bail it out.

3.3. Risk-minimizing bailout policy

Having derived risk-minimizing policies under each scenario, we are ready to combine them to characterize the risk-minimizing bailout policy. From Lemmas 3 and 4, we know that candidate risk-minimizing bailout policy $\beta(\gamma)$ should belong to the family

$$\beta(\dot{\gamma}) = \begin{cases} 1 & \text{if } \gamma \leqslant \dot{\gamma}, \\ 0 & \text{if } \gamma > \dot{\gamma}, \end{cases} \tag{23}$$

which can be fully characterized by the threshold state $\dot{\gamma}$. The following lemma characterizes the equilibrium level of bank risk and shows that it is a continuous function of this threshold $\dot{\gamma}$:

Lemma 5. (i) *The bank's optimal choice of risk is continuous in the threshold aggregate state* $\dot{\gamma}$.

(ii) *For any bailout policy* $\beta(\dot{\gamma})$, *a bank chooses to avoid default if, and only if,* $\dot{\gamma} \geqslant \gamma_n$.

(iii) *Risk is monotonically increasing in* $\dot{\gamma}$, *for* $\dot{\gamma} > \gamma_n$.

The lemma clearly illustrates the value effect of a bailout policy: A more generous policy (a larger threshold $\dot{\gamma}$) is associated with a higher bank value which, in turn, raises the stakes involved in the bank's risk-taking decision, inducing the bank to invest in safer assets. More importantly, it tells us that as we increase the threshold $\dot{\gamma}$ (and bank value), and get closer to the point γ_n at which the non-default condition just binds (part (ii) of the lemma), we may face two cases. If $\gamma_d \geqslant \gamma_n$, then the policy $\beta_n(\gamma)$ leads to the minimum attainable risk. Once bank's value is sufficiently high to ensure its survival, additional central bank assistance only makes risky investment decisions less costly, generating moral hazard without any counteracting value effect (part (iii)).[21] If, on the contrary, $\gamma_d < \gamma_n$, we reach the risk-minimizing policy $\beta_d(\gamma)$ at a point at which bank value is not large enough to prevent default. Extending the coverage to include higher values of γ would then have a negative impact on risk. Finally, the continuity with respect to the threshold $\dot{\gamma}$ ensures that no jump in equilibrium risk occurs at the point in which the non-default constraint is just binding, $\gamma_n = \dot{\gamma}$ (point (i)).

[21] Note that, if this is the case, the bank's probability of survival no longer depends on the threshold $\dot{\gamma}$. Indeed, additional central bank funds would be substituting for the shareholders' funds that would otherwise capitalize the bank.

This intuition is more formally stated in the following proposition.

Proposition 2. (i) *Bank risk is minimized by a policy* $\beta(\dot{\gamma}^{RM})$ *that bails out with certainty whenever the aggregate state is below a threshold state* $\dot{\gamma}^{RM}$, $\dot{\gamma}^{RM} = \min\{\gamma_d, \gamma_n\}$, *and lets the bank fail otherwise.*

(ii) *The equilibrium level of risk under this policy is always higher than the optimal,* X^*.

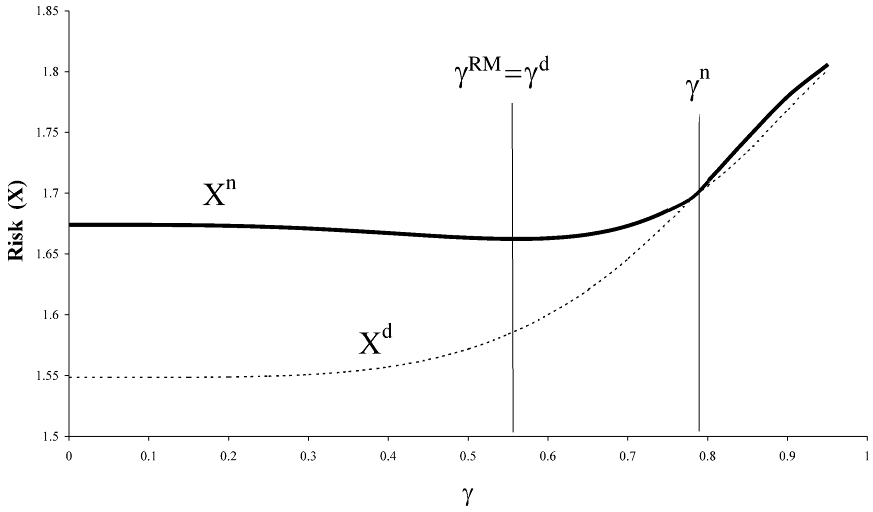

Fig. 1. Bank's risk choice ($\delta = 0.8$).

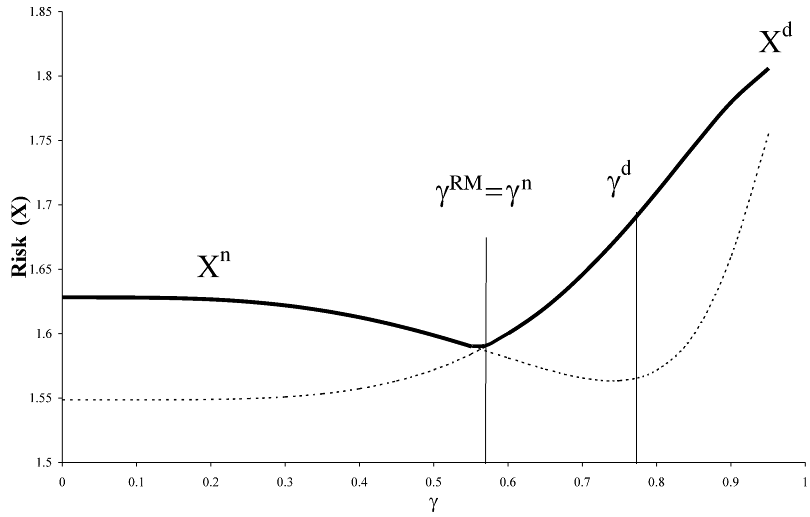

Fig. 2. Bank's risk choice ($\delta = 0.9$).

The two cases discussed above are illustrated in Figs. 1 and 2, where the level of risk chosen by the bank is plotted as a function of $\dot{\gamma}$.[22]

In Fig. 1, we have that $\gamma_n > \gamma_d$, so that the risk-minimizing bailout policy (denoted by superscript *RM*) is associated with the threshold $\gamma^{RM} = \gamma_d$, and a troubled bank defaults if not bailed out.

In Fig. 2, a higher value of δ induces bank shareholders to avoid default at a lower threshold level. As a result, $\gamma_n \leqslant \gamma_d$, and the risk-minimizing bailout policy is such that $\gamma^{RM} = \gamma_n$.

3.4. Central bank objective

From the previous discussion, it follows that the risk-minimizing bailout policy characterized in Proposition 2 is the one that brings the banks' choice of risk closer to the central planner's. Accordingly, if the central bank and the central planner share the same objective, such a policy would also be the one that maximizes the central bank's objective function.[23] However, one cannot rule out the possibility that, because of the presence of efficiency costs associated with the funding of the bailout policy, the central bank might also be concerned with the size of combined quasi-fiscal cost under different bailout schemes. By combined quasi-fiscal costs we refer both to the amount paid to insolvent banks if bailed out, and to the amount paid directly to depositors through the deposit insurance scheme, if the bank is liquidated.[24] In what follows, we illustrate how the introduction of such considerations in the central bank objective influences the nature of an 'optimal' bailout policy.

Before proceeding further, in order to simplify notation, and without loss of generality, we restrict out attention to policies of the type described in (23), denoting for simplicity any policy $\dot{\beta}(\gamma)$ by its threshold $\dot{\gamma}$. A central bank's objective function *CB* that weighs expected quasi-fiscal outlays $O(\cdot)$ by a factor $\psi \geqslant 0$ can be expressed as:

$$\max_{\dot{\gamma}} CB(\dot{\gamma}) \equiv \mu p\big[X(\dot{\gamma})\big]X(\dot{\gamma}) - \psi O(\dot{\gamma}), \tag{24}$$

with

$$O(\dot{\gamma}) \equiv r\int_0^{\overline{\gamma}} \big(1 - \gamma p\big[X(\dot{\gamma})\big]\big)\,\mathrm{d}\gamma, \quad \text{and} \quad \overline{\gamma} = \begin{cases} \dot{\gamma} & \text{if } \dot{\gamma} \geqslant \gamma_n, \\ 1 & \text{if } \dot{\gamma} < \gamma_n. \end{cases}$$

As in the previous section, we should distinguish between two possible cases, according to whether or not insolvent banks choose to avoid default if bailout money is forthcoming. In the first case ($\gamma_n < \gamma_d$), it is easy to verify that the risk-minimizing bailout policy $\dot{\gamma}^{RM} = \gamma_n$ also minimizes total outlays (Fig. 2). Any bailout policy $\dot{\gamma} < \gamma_n$ would not only

[22] The figures are plotted for a beta distribution function, with parameters $\nu = 4$, $\omega = 2$, assuming $p(X) = X + 0.5(1 - X^2)$, and $r_f = 1$. Figures 1 and 2 correspond to $\delta = 0.8$, and $\delta = 0.9$, respectively.

[23] This follows directly from the monotonicity of the V_c with respect to X, in (2).

[24] Thus, the analysis applies irrespective of whether the bailed out bank's liabilities are assumed by the deposit insurance fund or are paid for by bailout money provided directly to the bank by the central bank.

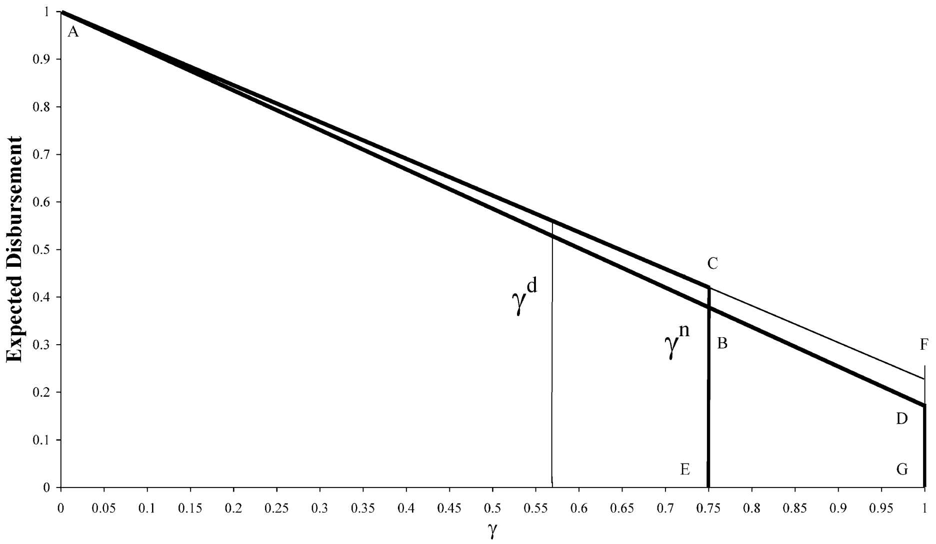

Fig. 3. Quasi-fiscal outlays ($\delta = 0.8$).

be associated with a higher level of risk, but it would also induce insolvent banks to default for $\gamma > \dot{\gamma}$, adding to the central bank's bill. On the other hand, a policy $\dot{\gamma} > \gamma_n$ would increase the banks' risk-taking incentives while committing central bank resources in situations in which bank shareholders would otherwise voluntarily produce the funds.

The second case ($\gamma_n > \gamma_d$) is more interesting. Here, under the risk-minimizing policy $\dot{\gamma}^{RM} = \gamma_d$, default is possible for $\gamma > \dot{\gamma}$. As is clear from a simple inspection of Fig. 1, by implementing the alternative policy $\dot{\gamma} = \gamma_n$, the central bank can transfer part of the expected financial loss to troubled banks that are now willing to recapitalize in those states of nature for which a bailout is not expected, at the cost of a higher risk level ($X(\gamma_n) > X(\gamma_d)$). The relevant trade-off is illustrated in Fig. 3.

Lines *AD* and *AF* represent expected outlays as a function of γ, $r(1 - \gamma p[X(\dot{\gamma})])$, for $\dot{\gamma} = \gamma_d, \gamma_n$, respectively. Note that, since the bank takes on more risk when the policy γ_n is implemented, the line *AF* is flatter than *AD*. By implementing a bailout policy γ_n, the central bank induces bank shareholders to recapitalize when $\gamma > \gamma_n$, and thus saves an amount equal to the area of the trapezoid *EBDG*. At the same time, for all $\gamma < \gamma_n$, the expected disbursement increases by the area of the triangle *ABC*. Furthermore, while the relative size of these areas depends on parameter values, as γ_d gets closer to γ_n, the central bank may find it optimal to go beyond the risk-minimizing policy if its priority is to reduce its quasi-fiscal outlays.

Summarizing our findings, the introduction of quasi-fiscal outlays in the objective function of the central bank, if anything, enlarges the set of states for which a bailout policy is optimal (increasing the threshold $\dot{\gamma}$). More formally, if we denote by $\dot{\gamma}^{CB}$ the central bank's optimal policy, we can claim that:

Proposition 3. *The bailout policy that solves the central bank problem* (24) *is never less generous than the risk-minimizing policy, that is* $\dot{\gamma}^{CB} \geqslant \dot{\gamma}^{RM}$.

As noted in the introduction, we deliberately abstracted so far from any bankruptcy costs associated with negative externalities arising from the failure of individual banks or with the loss of value due to bank liquidation. However, it is straightforward to show that the previous remark extends to the case in which these costs enter explicitly into the central bank's objective. Indeed, should this be the case, the optimal bailout policy would remain the same when $\gamma_n < \gamma_d$ (with their benefits enhanced by the reduction of the frequency of bank failures), whereas for $\gamma_n > \gamma_d$ it would increase the relative benefits of a more generous policy that, by inducing a 'bail in' from the bank's shareholders, eliminates defaults (and bankruptcy costs) altogether. In both cases, an active bailout policy induces the bank to take on a level of risk lower than the one it would have chosen in a *laissez faire* situation.

Last but not least, it is important to stress that our analysis implicitly assumed that the distortionary costs of bank bailouts are the same as those associated with the DIS, since, from a fiscal standpoint, both can be financed in identical manner. However this can very well not be the case. Indeed, if for some reason the latter are smaller than the former, then the optimal bailout policy may be less generous than the risk-minimizing policy. The same would be the case if, in absence of a deposit insurance scheme, the central bank gave an important weight to its quasi-fiscal outlays.

4. Discussion

Several important points deserve to be noted in relation to the previous results. First, the minimum risk attainable through a bailout scheme of the sort analyzed in the paper is an increasing function of the deposit rate. A simple inspection of (15) and (21) reveals that, for any given bailout function $\beta(\gamma)$, higher deposit rates are associated with smaller values of $\eta(X)$ (alternatively, higher equilibrium risk). This is not surprising since higher funding costs unambiguously detract from the bank's value, and thus increase risk-taking incentives. Thus, when deposit rates are low enough, $r < p(X_n(\dot{\gamma}))X_n(\dot{\gamma})/\psi(\dot{\gamma})$, the optimal bailout policy not only achieves a lower equilibrium risk level (thus reducing the frequency with which the central bank assumes bank losses) but also, by eliminating defaults altogether, transfers to the private bank the financial losses incurred in those states of nature for which the bailout policy is not activated. The same can be said of the discount rate $1/\delta$, since the present value of future rents, through which the value effect operates, is higher the lower the rate at which they are discounted.[25] In both cases, the impact on the optimal policy differs according to whether γ_d is below or above γ_n. For example, whereas

[25] The negative link between financing costs and discount rates, on the one side, and the choice of risk, on the other, is a natural implication of our basic model and does not depend on the existence of a bailout policy. In particular, it suggests that countries with higher costs of capital are more prone to engaging in excessive risk.

in the first case, the threshold increases with δ,[26] in the second an increase in δ has the opposite effect, since banks recapitalize under less generous policies, and the central bank can afford to limit its support while obtaining a lower level of risk at the same time.[27]

Second, the optimal bailout policy requires that in those states of nature for which the bailout scheme is activated ($\gamma \leqslant \dot{\gamma}$), funds should be made available with certainty ($\beta = 1$). Thus, in our framework a 'constructive ambiguity' approach that leaves the possibility of financial support in a given state of nature at the discretion of the central bank yields a higher equilibrium risk level than (at least some) non-discretionary policies that bail out failed banks in some states, and provide no support in the remaining ones. This is a logical corollary of the model, in which the relevant trade-off weights the incentives to reduce risk associated with the 'value' effect against the moral hazard effect of ensuring the continuance of the bank. This finding contrasts with the context in which the 'constructive ambiguity' approach is usually developed, where the moral hazard effect, the only channel through which the central bank is assumed to (negatively) influence risk-taking decisions, is weighted against potential losses arising from disruptions in the financial system and other bankruptcy costs.

A number of important (explicit and implicit) simplifying assumptions made in the paper deserve some further consideration. In what follows, we address them in turn.

4.1. Partial deposit insurance

The assumption of full deposit insurance, which simplified the derivation of the result by making the deposit rate r an exogenous variable, may be relaxed without altering the main findings of the paper. In the absence of insurance, risk-neutral depositors would demand a return on bank deposits equal to $r = r^f/E(s)$, where r^f is the return on the risk-free asset, and $E(s)$ denotes depositors' expectation of the probability that the deposit contract is honored.[28]

In this environment, a bailout policy would reduce the deposit rate both through its beneficial effect on portfolio risk (higher $p(X)$), as uninformed but rational depositors anticipate the impact of the policy on individual banks' decisions, and through an increase in the probability of repayment $s = p(X)(\mu - C) + \overline{\beta}$, which is higher under a bailout scheme for any level of portfolio risk. In turn, as discussed above, lower deposit rates would result in a second order positive effect on risk from the increase in value due to the widening of intermediation margins, amplifying the impact of the policy.

[26] It is easy to verify this by applying the implicit function theorem to (A.4) in Appendix A, from which we have that

$$\frac{\partial \gamma^d}{\partial \delta} = -\frac{\partial G}{\partial \delta} \Big/ \frac{\partial G}{\partial \gamma} = \frac{\int_{\gamma^d}^{1} \gamma f(\gamma)\,\mathrm{d}\gamma - \gamma^* \int_0^{\gamma^*} f(\gamma)\,\mathrm{d}\gamma}{1 - \delta \int_0^{\gamma^*} f(\gamma)\,\mathrm{d}\gamma} > 0.$$

[27] A lower threshold implies a lower value of C which, under the non-default scenario, leads to lower equilibrium risk.

[28] It is straightforward to show that the following argument holds when deposits are partially insured.

4.2. Information structure

Critical to our results is the proposed information structure, in particular our assumption that, while the portfolio choice is the bank's private information, the aggregate state γ is observed by the central bank. The first part is based on our understanding that, even if the regulator may have information about the composition of the investment portfolio of a troubled bank at the time emergency funds are requested, in practice this information tends to be rather vague and, in any case, difficult to assess objectively in order to be used as a condition for the provision of open assistance.

On the other hand, we interpret the aggregate state as a proxy for readily verifiable overall macroeconomic conditions beyond the influence of individual bank managers. It is important to recognize, however, that supervisors can often identify those conditions only imperfectly. In order to assess the extent to which the introduction of noisy signals on the realization of γ affects our results, consider a simplified setup in which γ is distributed according to the following two-point distribution:

$$\gamma = \begin{cases} \gamma_B & \text{with probability } \pi, \\ \gamma_G & \text{with probability } 1-\pi, \end{cases}$$

where the subscripts B and G denote 'bad' and 'good' states, respectively (i.e., $\gamma_B < \gamma_G$). In addition, assume that the central bank only observes an imperfect signal $\omega \in \{\omega_B; \omega_G\}$ of the true state of the nature, and that $\Pr(\omega_B \mid \gamma = \gamma_B) = \Pr(\omega_G \mid \gamma = \gamma_G) = \rho$, where $\rho \in [1/2; 1]$ denotes the precision of the signal so that a higher ρ corresponds to higher precision. Then, it can be shown that, for sufficiently bad states and sufficiently precise signals, the previous results can be extended to this case, as the following lemma formally states:

Remark 1. (i) For a large enough difference between good and bad states, that is when

$$\tilde{\gamma} \equiv \frac{\gamma_B}{\gamma_G} < 1 - \frac{1-\delta}{1-\delta\pi} \equiv \tilde{\gamma}^{\max}, \tag{25}$$

there exists a minimal level of precision ρ^* such that, for $\rho > \rho^*$, risk is minimized by a policy that bails out banks with certainty whenever a bad signal ω_B is observed, and lets banks fail otherwise.

(ii) Such minimal level of precision ρ^* first decreases and then increases with the frequency of bad states π.

(iii) Finally, ρ^* monotonically increases with the quality of the bad state of nature γ_B, and it monotonically decreases with the quality of the good state of nature γ_G, and with the discount factor δ.

The intuition for the first result is straightforward: The noisier the signal, the more likely it is that the moral hazard component of the bailout policy dominates the value effect. This is not surprising since, if the observed signal is extremely inaccurate, the bailout rule would be virtually uncorrelated with the realization of the actual shock, and thus it would come close in nature to a blanket policy that, as noted, only stimulates risk-taking behavior. Moreover, condition (25) also highlights that bad states should be sufficiently distant from the average state in order for the contingent rule to overcome the moral hazard associated with the possibility of a bailout.[29]

This condition underscores the second finding, namely, the fact that the minimal precision ρ^*, required for a risk-reducing bailout policy to exist, is non-monotonic in the frequency of bad states, π (first decreasing and then increasing with π). To grasp the intuition, it is helpful to note that bailing out banks following a signal ω_B ($\beta_B = 1$) minimizes risk if, and only if,

$$\Gamma \equiv \delta\mu - E(\gamma \mid \omega = \omega_B) > 0.^{30} \tag{26}$$

Thus, the scope for a risk-reducing bailout policy depends positively on the average state of nature, μ (fewer bailouts to greater value effect) and negatively on the average bailout state, $E(\gamma \mid \omega = \omega_L)$ (greater moral hazard due to frequent 'wrong' bailouts). An increase in the frequency of bad states π reduces both the terms and, for any precision level that satisfies (26), the difference between the first and second term first grows and then falls with π.

Finally, the last part of the proposition simply indicates that the higher is the difference between the two states the higher is the value effect associated with the bailout policy, and thus the lower the precision of the signal needed for a risk-reducing bailout policy to exist. The same argument holds for a decrease in the rate at which the bank discounts the future (higher values of δ).

Note that if banks' behavior has any impact on the value of the variable γ, as would be the case, for example, if the bailout scheme were activated whenever individual bank problems threatened to become systemic, then the policy would generate additional moral hazard on the part of large institutions that anticipate that their eventual failure may act as an automatic trigger for the provision of emergency funds. Furthermore, one of the reason why adverse macroeconomic shocks lead to multiple bank failures is that banks often have common exposures to such aggregate risk. A scheme like the one proposed in this paper would indeed increase banks' incentives to raise their exposure to those macroeconomic risks that would trigger the bailout policy. Accordingly, had we explicitly modeled the bank choice regarding its exposure to macroeconomic risk (γ), our conjecture is that the critical level of the aggregate shock $\dot{\gamma}$ below which the bailout is optimal would have been reduced. Also, as long as the amplitude of the shocks is difficult to measure, it is quite likely that the central bank will look at the performance of the rest of the economy (and particularly at that of other banks) when deciding whether to activate a bailout or not. Were this the case, herding problems as in Scharfstein and Stein (1991) and Rajan (1994) might also arise and increase the moral hazard effect of the bailout policy.

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[29] This can be seen more clearly by noting that (25) can be rewritten as $\gamma_B < \delta[(1-\pi)\gamma_G + \pi\gamma_B] = \delta\mu_N$.

[30] This follows from the fact that (A.5) and (A.6) in Appendix A can be expressed as

$$\frac{\partial\theta_N}{\partial\beta_i} < 0 \iff E(\gamma \mid \omega = \omega_i) < \frac{1}{\theta_N}, \quad \text{and}$$

$$\left.\frac{\partial\theta_N}{\partial\beta_B}\right|_{\beta_G=0,\beta_B=1} < 0 \iff E(\gamma \mid \omega = \omega_i) < \frac{1}{\delta\mu}.$$

The same is true if aggregate shocks are highly persistent, as banks may have incentives to engage in excessive risk-taking after a bad shock is observed to exploit the higher probability of being bailed out in the future due to persistent adverse conditions. However, it is realistic to assume that prudent bank practices involve long-term investments decisions that are not likely to be reversed overnight to profit from a higher bailout probability that at any rate is bound to be temporary.

Whereas an exact measure of the relevant aggregate factors influencing investment returns may be difficult to devise, one can think of a number of directly quantifiable variables affecting the cost of (short-term) financing or the repayment capacity of bank debtors, that can be used as signals of a sudden deterioration of the financial environment.[31] However, in reality one should not interpret the bailout scheme discussed here as a strict contract that specifies a unique value of any of these variables beyond which any assistance is denied, but rather as a guide for central bank intervention that calls for some unavoidable degree of discretion at the time of judging the current state of nature.

Finally, condition (26) stresses the fact that the rationale for the policy is intimately related to the presence of sufficiently bad states of nature, which in our stylized model was implicitly assumed by allowing for the possibility of $\gamma = 0$. This, in turn, yield important implications as to the kind of countries for which this policy is likely to be more valuable. Since the policy is framed in terms of a threshold state, countries with limited volatility that only rarely reach catastrophic states would seldom implement it. On the other hand, highly volatile countries (e.g., crisis-prone emerging economies subject to largely exogenous external shocks) will certainly benefit the most from explicit bailout policies as those described above, due both to the relative importance of more readily unidentifiable aggregate shocks and to the dispersion between good and bad states of nature, which facilitates conditioning.

4.3. Bank returns

In order to provide a simple illustration of an optimal bailout policy we dealt with a rather special structure of bank returns. In particular, we assumed:

(i) that increases in bank risk only affect the probability of default rather than the size of the losses when default occurs;
(ii) that expected returns are strictly concave in risk;
(iii) that the probability that insolvency is caused by the bank's imprudent risk behavior is proportional to the value of the shock γ.

The first two assumptions greatly simplify the analysis, but our results are robust to their relaxation. In fact, all we need is a set-up in which bank failures are possible and, because of limited liability, banks have incentives to take on more risk than socially optimal. In any such setup, a bailout policy that carefully balances its moral hazard and value effect

[31] Examples include negative real shocks depressing the level of economic activity, sudden changes in country risk that push up domestic interest rates, or sharp devaluations that increases the risk of default on foreign currency loans to producers of non-tradables.

components should lead to a reduction in the level of risk.[32] An extreme example can help clarify the intuition. Assume that, for some (catastrophic) states of nature, the probability of a bank failure for any given bank portfolio choice is sufficiently high. Then, the moral hazard component of a bailout policy becomes negligible and a commitment to bail out failed banks in those states can only reduce risk-taking incentives by increasing the bank's value. In other words, the qualitative features of the bailout policy do not depend on the structure of the idiosyncratic component of bank risk.

The third assumption guarantees that portfolio risk X satisfies Milgrom's (1981) monotone likelihood ratio property, which in the context of our model can be restated in the following way: For any two risk choices X_1 and X_2, with $X_1 > X_2$, and two states of nature γ_1 and γ_2, with $\gamma_1 > \gamma_2$, the relative likelihood that a failed bank chose X_1 (rather than X_2) is higher in state of γ_1 than in state γ_2. The property, standard in the moral hazard literature, ensures that the bailout rule is monotonic in $\dot{\gamma}$. The assumption intends to capture the nature of risk as a higher sensitivity to shocks,[33] and the fact that the line between safe and risky investments tends to blur when the economy is faced with large negative shocks that trigger a generalized collapse of asset prices or massive bankruptcy in the productive sector. Its relaxation, however, would only affect the monotonic nature of the bailout policy but not its very existence. For example, as the impact of a bailout policy on risk will no longer be monotonic in $\dot{\gamma}$, the risk-minimizing policy may involve assisting banks only for $\gamma \in [0, \gamma_1] \cup [\gamma_3, \gamma_4]$ (with $0 < \gamma_1 < \gamma_3 < \gamma_4$). At any rate, there still exists a policy that attains a lower risk level than that resulting from the absence of bailouts.

4.4. Alternative solutions

While our model provides a rationale for the implementation of state-contingent bailout policies, other alternative solutions to the banks' risk-shifting moral hazard addressed here may be conceived. In particular, banks may be able to hedge their portfolio risks at a fair price in the financial market by buying claims that pay off in adverse economic conditions—inasmuch as these conditions are verifiable. This would induce similar effects on incentives and would eliminate the need for bailouts. Another alternative solution to excessive risk taking that could a priori eliminate the need for bailouts is the imposition of capital requirements.[34] We discuss these two alternatives in turn, and argue that both such policies, while helpful in reducing risk-taking incentives, do not necessarily make our risk-reducing bailout policy redundant.

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[32] Consider, for instance, a set up *à la* Matutes and Vives (2000) in which banks choose between projects with the same expected returns, but different variance. In such a model, a bailout policy of the sort described in the previous section would induce banks to move away from maximum risk.

[33] Think, for example, of two perfectly correlated assets with different betas, where a higher beta will be associated with a higher level of risk.

[34] In addition, regulation targeted at bank mergers or minimum bank size may help limit risk taking, inasmuch as they contribute to increase banks' charter value.

4.4.1. Private bank insurance

It can be shown that, in the context of the present model, the risk-minimizing policy $\dot{\gamma}^{RM}$ characterized in Proposition 3 cannot be implemented privately through a fair insurance scheme. More precisely, it can be shown that the expected (per bank) outlays of any insurance policy characterized by the threshold $\dot{\gamma}^{RM}$ exceed the gains in terms of charter value due to the insurance policy, so that a fairly priced insurance contract is not feasible.[35] To see this, note that, in each time period, the expected insurance cost equals the gross deposit return r times the probability of an insurance claim that is given $[1-\gamma p(X(\dot{\gamma}))]\beta(\dot{\gamma})$, or, using (12), by $s(X(\dot{\gamma})) - p(X(\dot{\gamma}))\mu$, so that its expected (per bank) cost is:

$$T(\dot{\gamma}) \equiv r\frac{s(\cdot) - p(\cdot)\mu}{1 - \delta s(\cdot)}.$$

In turn, the value of the bank net of the (fair) insurance premium is given by

$$\Omega(\dot{\gamma}) \equiv \frac{p(\cdot)X(\dot{\gamma})\mu - rs(\cdot)}{1 - \delta s(\cdot)} > 0. \tag{27}$$

It is now easy to show that

Remark 2. No fairly priced insurance policy is feasible.

The finding that the same scheme is optimal for the central bank but not for the market simply reflects the fact the central bank does (and should) factor in additional pecuniary costs related with the failure of a bank. More precisely, in the context of our model, the insured bank does not take into account the cost associated with deposit insurance. Similarly, the presence of additional externalities (such as bankruptcy costs, contagion and domino effects) not incorporated in the bank's problem, work in the same direction, increasing the social vis à vis the private value of the policy.

4.4.2. Capital requirements

Interestingly, it can be shown that, for a large enough cost of capital, the beneficial effect of capital requirements on moral hazard is more than offset by their value effect. To see that, assume that capital requirements amount to a fraction k of total bank assets, so that, denoting the opportunity cost of capital by ϕ, the bank's value can now be written as

$$V_{CAR} \equiv \max_X \frac{\mu p(X)(X - (1-k)r) - \phi k}{1 - \delta\mu p(X)}. \tag{28}$$

Then it is easy to show that at any interior equilibrium:

Remark 3. If the opportunity cost of capital is large enough, that is when $\phi > r/\delta$, an increase in capital requirements increases risk-taking incentives.

[35] This finding is somehow reminiscent of Chan et al. (1992) fairly priced deposit insurance impossibility result.

The underlying intuition is the same as in Blum (1999). Higher values of k, by reducing banks' leverage, limit risk-taking incentives in the one shot game, they also reduce the 'value' of the bank and thus the incentives to behave more prudently, when the future is not too heavily discounted. It then turns out that, for large opportunity costs (alternatively, larger costs of capital), the latter effect dominates the former. Thus, capital requirements could be used to curb risk appetite only in those cases in which the cost of capital is not substantial, and thus they do not rule out the scope for a risk-reducing bailout policy.

5. Final remarks

Although our results seem to be consistent with the existing evidence from past bailouts, in that financial aid from the central bank to troubled banks are more likely in times of financial crises, our rationale is of a very different nature than those usually invoked to justify the standard LOLR practice. According to the latter, while an ex-ante commitment to assist troubled institutions induces moral hazard (increasing banks' risk appetite) and therefore should be avoided; discretional assistance is ex-post justified in light of the large costs associated with massive bank failures in crisis periods. Our analysis, instead, yields different empirical implications, predicting that the 'value effect' (that is, the incentives for prudent behavior induced by an increase in charter value) generated by a well-designed bailout policy more than offsets its moral-hazard component, reducing the risk appetite of banks.

According to this view,

(i) optimal policies should unambiguously commit to making funds available during periods of adverse macroeconomic conditions, as opposed to crises that are in itself intimately related with banks' past portfolio decisions; and
(ii) the central bank should intervene systematically (that is, as a rule) contingent on those conditions, either providing or withholding emergency funds, to all requesting banks.

While we deliberately chose to ignore potential bankruptcy costs in the core of the paper, these costs would at any rate make the provision of central bank emergency assistance even more desirable, as shown in the previous section and in line with the conventional view.

This paper may be extended in several interesting ways. For example, our analysis suggests that charging a penalty rate on central bank aid may limit the incentive effect of the policy as the burden of the debt reduces expected future rents. On the other hand, provided there are efficiency costs in the use of public money to ensure the continuance of insolvent banks, even if this insolvency was due solely to bad luck, the central bank may find it optimal to charge an 'access fee' to the bailout facility, in order to limit the associated quasi-fiscal cost. At any rate, there is an obvious constraint on the size of this fee, as shareholders of financial institutions in distress would agree to participate in a rescue plan only if expected future rents exceed the required contribution. If the bank's share of the burden is too costly, the institution may choose to follow an alternative strategy: Forced to make a killing or die, it may choose to gamble for resurrection, with the undesired effects of increasing risk-taking incentives and, by placing central bank money at risk, possibly increasing central bank losses as well. The threat of a punishment carries no weight if it is associated with a scenario in which the one who is being punished has nothing else to lose and penalties are ex-post unenforceable.

Since it is the subsidy component (the carrot) that reduces the banks' appetite for risk, any characteristic of the policy that detracts from the value of this subsidy conspires against this goal. In particular, should the central bank keep a claim on the bank's charter after the bailout, the associated value effect would weaken or, in the limit, vanish, and with it the beneficial impact of the policy on risk taking. The same conclusion is valid if the bailout is conducted in a way that compromises the bank's own reputation.[36]

A related argument applies if we consider the case, deliberately ignored in the previous discussion, of a central bank that cannot commit the necessary resources for a complete bailout, as is typical in economies where financial intermediation is largely conducted in a foreign currency of which the central bank has only limited holdings. If bank managers anticipate incomplete bailouts (that is, bailouts that demand a proportional contribution from the troubled bank's shareholders) in the event of a sudden worsening of macroeconomic conditions, the beneficial effect emphasized in the paper may be reduced. The accumulation of international reserves can certainly limit this possibility, but this solution does not come without costs. This suggests one way in which the present model can be extended to an international context. Inasmuch as aggregate shocks are not perfectly correlated across countries, an international insurer could provide similar 'insurance services' at a much lower cost in terms of the required reserves. Indeed, given that these policies are unlikely to materialize privately, as indicated by the finding that insurance costs exceed private gains in value, our results suggest an international lender of last resort that ensures that liquidity-constrained central banks can implement the optimal bailout policy in full could contribute to reduce bank risk in the long run.

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[36] The impact is analogous to that of a bank closure on the reputation of the supervisor, as in Boot and Thakor (1993). For example, in a partial-deposit-insurance-cum-heterogeneous-banks setup, the extension of emergency assistance, if observable by the market, could be interpreted as a (noisy) signal of a risky type, reducing (but not eliminating) the bailed out bank's future rents.

Appendix A. The bank's maximization problem in different scenarios

No bailout scenario

$$\text{Problem:}\quad \max_X v_a(X) \equiv \frac{(X-r)p(X)\mu}{1-\delta p(X)\mu};$$

$$\text{FOC:}\quad v_a'(X) = \mu\frac{(X-r)p'(X)+p(X)(1-\delta p(X)\mu)}{(1-\delta p(X)\mu)^2} = 0;$$

$$\text{SOC:}\quad v_a''(X) = \mu\frac{(X-r)p''(X)+p'(X)(2-\delta\mu)}{(1-\delta p(X)\mu)^2} + \frac{2\delta\mu^2 p'(X)\text{FOC}}{(1-\delta p(X)\mu)^3}.$$

When the FOC is equal to zero, the SOC is always negative ($2 > \delta\mu$). Since the bank's objective function is continuous and differentiable, this implies that if a critical point exists, it is unique and is a global maximum. Existence follows directly from the fact that the FOC is positive at $X = \underline{X}$, and negative at $X = \overline{X}$.

Default scenario

$$\text{Problem:}\quad \max_X v_d(X) \equiv \frac{p(X)(X-r)\mu}{1-\delta(p(X)(\mu-C)+\overline{\beta})};$$

$$\text{FOC:}\quad v_d'(X) = \mu\frac{(X-r)(1-\delta\overline{\beta})p'(X)+p(X)(1-\delta(p(X)(\mu-C)+\overline{\beta}))}{(1-\delta(p(X)(\mu-C)+\overline{\beta}))^2} = 0;$$

$$\text{SOC:}\quad v_d''(X) = \mu\frac{(X-r)(1-\delta\overline{\beta})p''(X)+2p'(1-\delta(p(X)(\mu-C)+\overline{\beta}))}{[1-\delta(p(X)(\mu-C)+\overline{\beta})]^2} + \frac{2\delta p' p(X)(\mu-C)\text{FOC}}{(1-\delta(p(X)(\mu-C)+\overline{\beta}))^3}.$$

When the FOC is equal to zero, the SOC is always negative. The same argument as in the previous scenario applies here.

Non-default scenario

$$\text{Problem:}\quad \max_X v_n \equiv \frac{\mu p(X)X - r(1+pC-\overline{\beta})}{1-\delta};$$

$$\text{FOC:}\quad v_n'(X) = \frac{\mu p(X)+p'(X)(\mu X - rC)}{1-\delta} = 0;$$

$$\text{SOC:}\quad v_n''(X) = \frac{p''(X)(\mu X - rC)+2\mu p'(X)}{1-\delta}.$$

Again, when the FOC is equal to zero, the SOC is always negative, and the same argument as in the previous scenarios applies here.

Proof of Lemma 2. $\operatorname{cov}(\beta(\gamma),\gamma) = \int_0^1(\gamma-\mu)(\beta(\gamma)-\overline{\beta})f(\gamma)\,\mathrm{d}\gamma = \int_0^1\gamma\beta(\gamma)f(\gamma)\,\mathrm{d}\gamma - \int_0^1\gamma\overline{\beta}f(\gamma)\,\mathrm{d}\gamma - \int_0^1\mu\beta(\gamma)f(\gamma)d\gamma + \mu\overline{\beta} = C - \mu\overline{\beta}$.
Finally, $\mu\delta\overline{\beta} > C \Rightarrow C - \mu\overline{\beta} < 0$. □

Proof of Lemma 3. First note that to find the bailout policy that minimizes the value of X that solves (15), it suffices to find the bailout function $\beta(\gamma)$ that minimizes θ_d.

By definition of the Stieltjes integral, we have that

$$\overline{\beta} = \int_0^1 \beta(\gamma) f(\gamma)\,\mathrm{d}\gamma = \lim_{\|\Delta\|\to 0} \sum_{k=1}^{n} \beta(\xi_k)\bigl[f(x_k) - f(x_{k-1})\bigr],$$

and

$$C = \int_0^1 \gamma\beta(\gamma) f(\gamma)\,\mathrm{d}\gamma = \lim_{\|\Delta\|\to 0} \sum_{k=1}^{n} \gamma_{\xi_k}\beta(\xi_k)\bigl[f(x_k) - f(x_{k-1})\bigr],$$

with

$$0 = x_0 < x_1 < \cdots < x_n = 1, \quad \|\Delta\| = \max(x_1 - x_0, x_2 - x_1, \ldots, x_n - x_{n-1}),$$
$$\text{and} \quad x_{k-1} \leqslant \xi_k \leqslant x_k.$$

Let us define:

$$\dot{\theta} \equiv \frac{1 - \delta\sum_{k=1}^{n}\beta(\xi_k)[f(x_k) - f(x_{k-1})]}{\delta(\mu - \sum_{k=1}^{n}\gamma_{\xi_k}\beta(\xi_k)[f(x_k) - f(x_{k-1})])},$$

noting that

$$\theta_d = \lim_{\|\Delta\|\to 0}\dot{\theta}.$$

It is easy to check the following:

$$\frac{\partial\dot{\theta}}{\partial\beta(\xi_k)} < 0 \quad \Leftrightarrow \quad \gamma_{\xi_k} < \frac{\delta\bigl(\mu - \sum_{k=1}^{n}\gamma_{\xi_k}\beta(\xi_k)[f(x_k) - f(x_{k-1})]\bigr)}{1 - \delta(\sum_{k=1}^{n}\beta(\xi_k)[f(x_k) - f(x_{k-1})]} = \frac{1}{\dot{\theta}}.$$

Since this is true for any arbitrary γ_{ξ_k} in any arbitrary partition, defining $\hat{\gamma}_k \equiv \lim_{\|\Delta\|\to 0}\gamma_{\xi_k}$, and $\gamma_d \equiv \lim_{\|\Delta\|\to 0} 1/\dot{\theta}$ we have that

$$\frac{\partial\dot{\theta}}{\partial\beta(\hat{\gamma}_k)} < 0 \quad \Leftrightarrow \quad \hat{\gamma}_k < \gamma_d. \tag{A.1}$$

This in turn implies that the bailout policy that minimizes θ, and in turn, maximizes the equilibrium effort can be characterized as

$$\beta_d(\gamma) \equiv \begin{cases} 1 & \text{if } \gamma < \gamma_d, \\ 0 & \text{if } \gamma > \gamma_d. \end{cases} \tag{A.2}$$

If the optimal bailout policy $\beta_d(\gamma)$ is implemented, we have that

$$\overline{\beta}\bigl(\beta_d(\gamma)\bigr) \equiv \int_0^{\gamma_d} f(\gamma)\,\mathrm{d}\gamma, \qquad C\bigl(\beta_d(\gamma)\bigr) \equiv \int_0^{\gamma_d} \gamma f(\gamma)\,\mathrm{d}\gamma,$$

and

$$\theta_d\big(\beta_d(\gamma)\big) = \frac{1-\delta\int_0^{\gamma_d} f(\gamma)\,\mathrm{d}\gamma}{\delta(\mu-\int_0^{\gamma_d}\gamma f(\gamma)\,\mathrm{d}\gamma)} = \frac{1-\delta\int_0^{\gamma_d} f(\gamma)\,\mathrm{d}\gamma}{\delta\int_{\gamma_d}^1 \gamma f(\gamma)\,\mathrm{d}\gamma}. \tag{A.3}$$

It remains to be proven that there is a unique $\gamma_d \in (0,1)$. From (A.3), γ_d must satisfy

$$\gamma_d = \frac{\delta\int_{\gamma_d}^1 \gamma f(\gamma)\,\mathrm{d}\gamma}{1-\delta\int_0^{\gamma_d} f(\gamma)\,\mathrm{d}\gamma},$$

or

$$G(\gamma) \equiv \gamma_d - \delta\int_{\gamma_d}^{1} \gamma f(\gamma)\,\mathrm{d}\gamma - \delta\gamma_d\int_0^{\gamma_d} f(\gamma)\,\mathrm{d}\gamma = 0. \tag{A.4}$$

Since $G(\gamma)$ is continuous in γ, $G(0) = -\delta\mu < 0$, and $G(1) = 1-\delta > 0$, there is at least one $\gamma_d \in (0,1)$ such that $G(\gamma_d) = 0$. Finally, the uniqueness of γ_d is ensured by the fact that $\partial G(\gamma)/\partial\gamma_d = 1-\delta\int_0^{\gamma_d} f(\gamma)\,\mathrm{d}\gamma > 0$. □

Proof of Lemma 4. We proceed in steps.

(i) First, we show that the risk-minimizing bailout policy belongs to the family of bailout policies

$$\beta(\dot\gamma) = \begin{cases} 1 & \text{if } \gamma \leqslant \dot\gamma, \\ 0 & \text{if } \gamma > \dot\gamma. \end{cases}$$

Notice, since from (21) risk is increasing in C, a necessary condition for a bailout policy to be risk-minimizing is that no other feasible policy exists (i.e., such that $r \leqslant \delta V_n = (\mu p(X_n)X_n - r(1+pC-\overline{\beta}))/(1-\delta)$) leading to a lower level of risk. In particular, since $\partial V_n/\partial C < 0$, if $\hat\beta(\gamma)$ is a risk-minimizing bailout policy, no other policy $\tilde\beta(\gamma)$ should exist such that $\overline{\beta}(\tilde\beta(\gamma)) = \overline{\beta}(\hat\beta(\gamma))$, and $C(\tilde\beta(\gamma)) < C(\hat\beta(\gamma))$. Assume that a bailout policy $\hat\beta(\gamma)$ with $\hat\beta(\gamma) \in (0,1)$ for $\gamma \in [\hat\gamma^-, \widehat{\gamma}^+]$ is a risk-minimizing bailout policy. Consider now a bailout policy $\tilde\beta(\gamma)$ with $\tilde\beta(\gamma) = \hat\beta(\gamma)$ for $\gamma \notin [\hat\gamma^-, \hat\gamma^+]$, $\tilde\beta(\gamma) = 1$ for $\gamma \in [\hat\gamma^-, \gamma^T]$, and $\tilde\beta(\gamma) = 0$ for $\gamma \in [\gamma^T, \hat\gamma^+]$, with $\gamma^T = \{\gamma\colon \overline{\beta}(\tilde\beta(\gamma)) = \overline{\beta}(\hat\beta(\gamma)\}$. Since $C(\tilde\beta(\gamma)) - C(\hat\beta(\gamma)) = \mathrm{cov}(\tilde\beta(\gamma),\gamma) - \mathrm{cov}(\hat\beta(\gamma),\gamma)$, and, by construction of $\hat\beta(\gamma)$, $\mathrm{cov}(\tilde\beta(\gamma),\gamma) < \mathrm{cov}(\hat\beta(\gamma),\gamma)$, we have that $C(\tilde\beta(\gamma)) < C(\hat\beta(\gamma))$, a contradiction. A similar argument proves that no bailout policy such that $\beta(\gamma) = 0$ for $\gamma < \dot\gamma$, and $\beta(\gamma) = 0$ for $\gamma > \dot\gamma$ can be a risk-minimizing bailout policy.

(ii) Second, we show that the equilibrium level of risk $X_n(\dot\gamma)$ increases with the bailout threshold $\dot\gamma$, and is always greater than X^*. In fact, since $\partial C(\dot\gamma)/\partial\dot\gamma = \gamma f(\gamma) > 0$, from (4), and (21), we have that $\partial X_n(\dot\gamma)/\partial\dot\gamma > 0$.

From (i) and (ii) it then follows that, in the non-default scenario, the optimal bailout policy is given by $\beta_n(\gamma)$. □

Proof of Lemma 5. We proceed by steps. First we show that the value of the bank increases in $\dot\gamma$, for all $\dot\gamma \in [0,1]$. For $\dot\gamma < \gamma_d$ (default scenario), differentiating (14) with respect to $\dot\gamma$ (for any given X, and thus a fortiori when we allow the bank to adjust its choice of risk) yields:

$$\frac{\partial V_d}{\partial \dot{\gamma}} = \frac{\delta p(X)(X-r)\mu}{[1-\delta(p(X)(\mu-C)+\overline{\beta})]^2}\big[f(\gamma)-p(X)\gamma f(\gamma)\big] > 0.$$

Similarly, for $\dot{\gamma} > \gamma_n$ (non-default scenario), differentiating (18) with respect to $\dot{\gamma}$

$$\frac{\partial V_n}{\partial \dot{\gamma}} = \frac{r}{1-\delta}\big[f(\gamma)-p(X)\gamma f(\gamma)\big] > 0.$$

By definition of γ_n, we have that $r = p(X(\gamma_n))X_n(\gamma_n)/\psi(\gamma_n)$. Substituting this value into (21) and (15), and rearranging

$$\lim_{\dot{\gamma}\to\gamma_n}\eta(X) = \lim_{\dot{\gamma}\to\gamma_d}\eta(X) = -1 - \frac{\delta C(\gamma_n)p(X(\gamma_n))}{1-\delta\bar{\beta}(\gamma_n)\mu p(X(\gamma_n))},$$

which implies that $V_n = V_d$ at $\dot{\gamma} = \gamma_n$, and thus that the optimal choice of risk is continuous in $\dot{\gamma}$ which proves part (i) of the lemma. The proof of part (ii) follows directly from the fact that $\delta V_n \gtreqless r$ iff $\dot{\gamma} \gtreqless \gamma_n$. Finally, the fact that risk is monotonically increasing in $\dot{\gamma}$, for $\dot{\gamma} > \gamma_n$, part (iii) of the lemma, follows from the fact that the risk choice is continuous in $\dot{\gamma}$, and from the proof of Lemma 4. □

Proof of Proposition 2. (i) By Lemma 4, among all policies $\beta(\dot{\gamma})$ with $\dot{\gamma} > \gamma_n$, the policy associated with the minimum risk is $\beta_n(\gamma)$. Then, for $\gamma_d > \gamma_n$, $X_n(\gamma_d) > X_n(\gamma_n)$. On the other hand, if $\gamma_d \leqslant \gamma_n$, we know that $X_d(\gamma_d) \leqslant X_d(\dot{\gamma})$, for all $\dot{\gamma} \geqslant \gamma_d$ and, in particular by Lemma 5, part (i), $X_d(\gamma_d) \leqslant X_d(\gamma_n) = X_n(\gamma_n)$.

(ii) If $\gamma_d > \gamma_n$, it follows directly from (3), (4), and (21) that

$$-1 + \frac{p'(X)C(\gamma_n)r}{\mu p(X)} < -1 \quad \Rightarrow \quad X_n \geqslant X^*.$$

If $\gamma_d \leqslant \gamma_n$, we know that

$$\psi(\gamma_d) = \theta(\gamma_d) - \frac{C(\gamma_d)(\theta(\gamma_d)-p(X_d))}{\mu} > \theta(\gamma_d),$$

from which $r \geqslant p(X_d)X_d/\psi > p(X_d)X_d/\theta_d$, and substituting into (15), $\eta(X_d) < -1$. □

Proof of Remark 1. Following the same steps as in the previous case, it is straightforward to show that the bailout policy that minimizes risk is the one that minimizes $\theta_N = (1-\delta\overline{\beta}_N)/\delta(\mu_N - C_N)$, with

$$\begin{aligned}
&\mu_N \equiv \gamma_B\pi + \gamma_G(1-\pi),\\
&\overline{\beta}_N \equiv \pi\rho\beta_B + \pi(1-\rho)\beta_G + (1-\pi)\rho\beta_G + (1-\pi)(1-\rho)\beta_B,\\
&C_N \equiv \gamma_B\pi\rho\beta_B + \gamma_B\pi(1-\rho)\beta_G + \gamma_G(1-\pi)\rho\beta_G + \gamma_G(1-\pi)(1-\rho)\beta_B,\\
&\beta_i = \beta(\omega_i), \quad i = B, G,
\end{aligned}$$

and the subscript N, denoting this new scenario. Differentiating θ_N with respect to β_i, we have

$$\frac{\partial\theta_N}{\partial\beta_G} < 0 \quad \Longleftrightarrow \quad \frac{(\gamma_B\pi(1-\rho)+\gamma_G(1-\pi)\rho)}{(\pi(1-\rho)+(1-\pi)\rho)} < \frac{(\mu_N - C_N)\delta}{(1-\delta\overline{\beta}_N)} = \frac{1}{\theta_N}, \tag{A.5}$$

$$\frac{\partial\theta_N}{\partial\beta_B} < 0 \quad\Longleftrightarrow\quad \frac{(\gamma_B\pi\rho + \gamma_G(1-\pi)(1-\rho))}{(\pi\rho + (1-\rho)(1-\pi))} < \frac{(\mu_N - C_N)\delta}{(1-\delta\overline{\beta}_N)} = \frac{1}{\theta_N}. \tag{A.6}$$

From (A.5) and (A.6) it follows that candidate risk-minimizing bailout policies have to satisfy $\beta_i \in \{0,1\}$, $i = G, N$. However, as long as the signal is informative ($\rho > 1/2$), it is easy to prove, following the steps of Lemma 2, that any risk-reducing bailout policy should satisfy $\beta(\omega_G) < \beta(\omega_B)$. It then follows directly from (A.6) that a necessary and sufficient condition for the only remaining candidate bailout policy ($\beta_G = 0$; $\beta_B = 1$) to be risk-reducing is given by

$$1 > \rho > \rho^* \equiv \frac{(1-\pi)[\gamma_G(1-\delta(1-\pi)) - \delta\gamma_B\pi]}{\gamma_G(1-\pi)[1-\delta(1-2\pi)] - \gamma_B\pi[1+\delta(1-2\pi)]},$$

which requires that

$$\tilde{\gamma} \equiv \frac{\gamma_B}{\gamma_G} < 1 - \frac{1-\delta}{1-\delta\pi} \equiv \tilde{\gamma}^{\max}. \tag{A.7}$$

Finally, differentiating ρ^* with respect to π, we have that

$$\frac{\partial\rho^*}{\partial\pi} = \frac{(1-\delta)(\gamma_B\gamma_G - \delta(-(1-\pi)\gamma_G - \gamma_B\pi)^2)}{(\pi\gamma_B(1+\delta(1-2\pi)) - \gamma_G(1-\pi)(1-\delta(1-2\pi))^2},$$

from which $\operatorname{sign}(\partial\rho^*/\partial\pi) = \operatorname{sign}[\Psi \equiv \tilde{\gamma} - \delta(1-(1-\tilde{\gamma})\pi)^2]$. The result that there is $\tilde{\pi} \in (0,1)$ such that $\partial\rho^*/\partial\pi > 0 \Leftrightarrow \pi > \tilde{\pi}$ follows directly from the fact that Ψ is quadratic in $\tilde{\gamma}$, $\Psi|_{\pi=0} = \tilde{\gamma} - \delta < 0$ (because of (A.7)), and $\Psi|_{\pi=1} = \tilde{\gamma} - \delta\tilde{\gamma}^2 > 0$. Finally, it can be readily verified that:

$$\frac{\partial\rho^*}{\partial\gamma_G} = \frac{-\pi\gamma_B(1-\delta)(1-\pi)}{(\pi\gamma_B(1+\delta(1-2\pi)) - \gamma_G(1-\pi)(1-\delta(1-2\pi))^2} < 0,$$

$$\frac{\partial\rho^*}{\partial\gamma_B} = \frac{\pi\gamma_G(1-\delta)(1-\pi)}{(\pi\gamma_B(1+\delta(1-2\pi)) - \gamma_G(1-\pi)(1-\delta(1-2\pi))^2} > 0,$$

$$\frac{\partial\rho^*}{\partial\delta} = \frac{-(\gamma_G-\gamma_B)(1-\pi)\pi(\gamma_G(1-\pi)+\gamma_B\pi)}{(\pi\gamma_B(1+\delta(1-2\pi)) - \gamma_G(1-\pi)(1-\delta(1-2\pi))^2} < 0. \qquad \square$$

Proof of Remark 2. To prove that no fair insurance policy is feasible, it suffices to show that the net value of the bank $\Omega(\dot{\gamma})$ is monotonically decreasing in $\dot{\gamma}$. Differentiating (27) with respect to $\dot{\gamma}$, we obtain that

$$\frac{\partial\Omega}{\partial\dot{\gamma}} = \frac{\partial X}{\partial\dot{\gamma}}\frac{1}{1-\delta s(\cdot)}\left\{\big(p'(\cdot)X(\cdot)+p(\cdot)\big)\mu - rs'(\cdot) + \frac{\delta s'(\cdot)[p(\cdot)X(\cdot)\mu - rs(\cdot)]}{(1-\delta s(\cdot))}\right\}. \tag{A.8}$$

We already know that, for $\dot{\gamma} \in [0, \dot{\gamma}^{RM}]$, risk X is decreasing in $\dot{\gamma}$. Thus, to prove the remark, it is enough to show that the expression into brackets in (A.8) is positive, or that

$$\big(p'(\cdot)X(\cdot)+p(\cdot)\big)\mu - rs'(\cdot) + \frac{\delta s'(\cdot)[p(\cdot)X(\cdot)\mu - rs(\cdot)]}{1-\delta s(\cdot)} > 0.$$

Finally, from (15) and $\theta < \theta_a = 1/\delta\mu$, we have that

$$-\left(1 - \frac{p(\cdot)X(\cdot)}{\theta}\right) = \frac{p'(\cdot)}{p(\cdot)}\big[X(\cdot) - r\big] < \frac{p'(\cdot)}{p(\cdot)}\left[X(\cdot) - \frac{r}{\delta\mu\theta}\right],$$

and, since $X > 1$,

$$\frac{p'(\cdot)}{p(\cdot)}\left[X(\cdot) - \frac{r}{\delta\mu\theta}\right] + \left[1 - \frac{p(\cdot)}{\theta}\right] > 0. \tag{A.9}$$

Substituting $\theta = \delta(\mu - C)/(1 - \delta\overline{\beta})$, rearranging and slightly manipulating, condition (A.9) can be written as

$$\big(p'(\cdot)X(\cdot) + p\big)\mu + \frac{p'(\cdot)(\mu - C)(\delta p(\cdot)X(\cdot)\mu - r)}{1 - \delta p(\cdot)(\mu - C) - \delta\overline{\beta}} > 0.$$

From which, using $s = p(\mu - C) + \overline{\beta}$ and $s' = p'(\mu - C)$, we get

$$\big(p'(\cdot)X(\cdot) + p(\cdot)\big)\mu + \frac{s'(\cdot)(\delta p(\cdot)X(\cdot)\mu - r)}{1 - \delta s(\cdot)} > 0,$$

which in turns implies (since $s < 1$, and $s'(\cdot) < 0$) that

$$\big(p'(\cdot)X(\cdot) + p(\cdot)\big)\mu + \frac{\delta s'(\cdot)[p(\cdot)X(\cdot)\mu - rs(\cdot)]}{1 - \delta s(\cdot)} > 0. \qquad \square$$

Proof of Remark 3. Differentiating (28) with respect to X it is easy to check that the first-order condition can be expressed as

$$G(X;k) \equiv (X - r)p'(\cdot) + (1 - \delta p\mu)p - kp'(\cdot)(\delta\phi - r) = 0, \tag{A.10}$$

from which

$$\frac{\partial G(X;k)}{\partial k} < 0 \quad \Longleftrightarrow \quad \phi < r/\delta.$$

In turn, charter value (28) should be positive in equilibrium, from which it follows that

$$\mu p(\cdot)\big(X - (1 - k)r\big) - \phi k > 0, \tag{A.11}$$

which in turn implies that any interior equilibrium satisfies

$$\frac{\partial G(X;k)}{\partial X} = 2p'(\cdot)\big[1 - \delta\mu p(\cdot)\big] + p''(\cdot)\big[X - (1 - k)r - k\delta\phi\big] < 0.$$

Moreover, it can be readily seen that $\partial G(X;k)/\partial X|_{G(X;k)=0} < 0$ is a necessary and sufficient condition for the second order condition to be verified. Finally, differentiating implicitly

$$\frac{\mathrm{d}X^*}{\mathrm{d}k} = -\frac{\partial G(X;k)}{\partial X}\bigg/\frac{\partial G(X;k)}{\partial X} > 0 \quad \Longleftrightarrow \quad \phi > r/\delta. \qquad \square$$

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